# Robust Statistics in IceCube Initial Muon Reconstruction

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**Abstract:** In the IceCube Neutrino Detector, muon tracks are reconstructed from the muon's light emission. The initial track reconstruction serves as a starting point for more sophisticated track fitting, using detailed knowledge of the ice and the detector. We describe here a substantial improvement of the accuracy in the initial track reconstruction for muons. Our approach is to couple simple physical models with robust statistical techniques. Using the metric of median angular accuracy, a standard metric for track reconstruction, this solution improves the accuracy in the reconstructed direction by 13%.

**Keywords:** Icecube, Muons, Track Reconstruction.

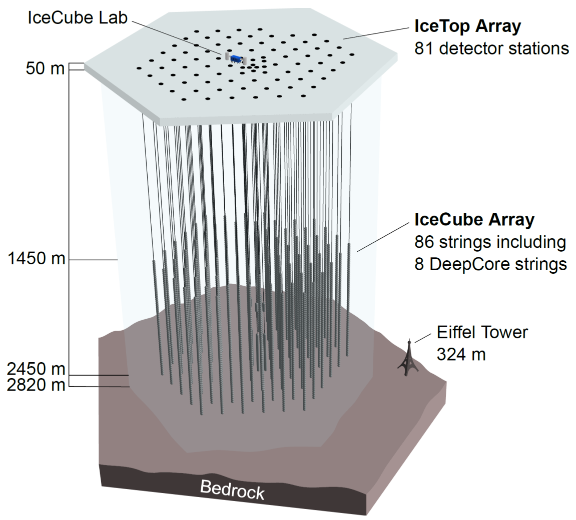


**Fig. 1**: The IceCube neutrino detector in the Antarctic ice. A picture of the Eiffel Tower is shown for scale.

## 1 Introduction

The IceCube neutrino detector searches for neutrinos that are generated by the universe's most violent astrophysical events: exploding stars, gamma ray bursts, and cataclysmic phenomena involving black holes and neutron stars [4]. The detector, roughly one cubic kilometer in size, is located near the geographic South Pole and is buried to a depth of about 2.5 km in the Antarctic ice [6]. The detector is illustrated in Figure 1 and a more complete description is given in Section 2.

This manuscript describes an improvement in the reconstruction algorithm used to generate the initial track reconstruction of detected muons in the IceCube detector. We achieve this improvement in accuracy using robust statistical techniques.

## 2 Background

The IceCube detector is composed of 5,160 optical detectors, each composed of a photomultiplier tube (PMT) and onboard digitizer [8]. The PMTs are spread over 86 vertical strings arranged in a hexagonal shape, with a total instrumented volume of approximately one cubic kilometer. The PMTs on a given string are separated vertically by 17 m, and the string-to-string separation is roughly 125 m.

When a neutrino enters the telescope, it sometimes interacts with the ice and generates a muon. As the muon travels though the detector, it radiates light [10], which is observed by the PMTs and broken down into discrete *hits* [7]. A collection of hits is called an *event*, and when the number of hits in an event is sufficiently large, this triggers the muon track reconstruction algorithm.

### 2.1 Cosmic Ray Muons

In addition to neutrinos, muons can also be generated by cosmic rays. The detector separates neutrino muons from cosmic ray muons by reconstructing the muon track and determining whether the muon was traveling downwards into the Earth or upwards out of the Earth. Since neutrinos can penetrate the Earth but cosmic ray muons cannot, it follows that a muon traveling out of the Earth must have been generated by a neutrino. Thus, by selecting only the muons that are reconstructed as up-going, the neutrino muons can, in principle, be isolated.

While separation is possible in principle, the number of observed cosmic ray muons exceeds the number of observed neutrino muons by five orders of magnitude [5]. Thus, high-accuracy reconstructions are critical for preventing erroneously reconstructed cosmic ray muons from dominating the neutrino analysis.

### 2.2 Challenges in Neutrino Detection

There are several challenges for the reconstruction algorithms used in the detector.

**Modeling Difficulties** The underlying physics of the system are nontrivial to model. The muon's light is scattered by the dust and air crystals in the ice medium. This scattering is both complex and stochastic, and the scattering properties of the ice vary with depth [11]. These challenges make it difficult to design a complete model of the muon's light scattering.

**Noise** The outliers inherent in the data present an additional challenge. The PMTs are so sensitive to light that they can record hits even in the absence of nearby muons. These hits arise from photons generated by the radioactive decay of the glass inside the PMT [9].

**Computational Constraints** Reconstruction algorithms need to be efficient enough to process about 3,000 muons per second with the computing resources available at the South Pole. Thus, algorithms with excessive computational demands are discouraged.

### 2.3 Prior IceCube Software

Starting with the positions and times of each hit, the detector reconstructs the muon track. After collecting the initial data, the data passes though a series of filters that removes hits isolated in space and time [1].

After removing outliers, the data is processed using a simple reconstruction algorithm, *linefit*, which finds the track that minimizes the sum of the squares of the distances between the track and the hits. More formally, assume there are $N$ hits; denote the position and time of the $i^{th}$ hit as $(\vec{x}_i, t_i) \in \mathbb{R}^3 \times \mathbb{R}$. Let the muon have a reconstructed velocity of $\vec{v}$, and let $(\vec{x}_0, t_0)$ be a point on the reconstructed track. The linefit reconstruction solves the *least-squares* optimization problem

$$\min_{t_0,\vec{x}_0,\vec{v}} \sum_{i=1}^{N} \rho_i(t_0,\vec{x}_0,\vec{v})^2, \tag{1}$$

where

$$\rho_i(t_0,\vec{x}_0,\vec{v}) = \|\vec{v}(t_i - t_0) + \vec{x}_0 - \vec{x}_i\|_2 . \tag{2}$$

The linefit reconstruction is primarily used to generate an initial track or *seed* for a more sophisticated reconstruction.

The reconstruction algorithm for the sophisticated reconstruction is *Single-Photo-Electron-Fit (SPE)* [5]. SPE takes the least-squares reconstruction and event data, and uses a likelihood maximization algorithm to reconstruct the muon track. The SPE reconstruction typically takes about two orders of magnitude longer to compute than linefit. The complete reconstruction process is outlined in Figure 2.

## 3 Our Improvements to Muon Track Reconstruction

We now discuss the improvements we make to the reconstruction algorithm. By augmenting the reconstruction algorithm using robust data analysis techniques, we show improvement in the reconstruction algorithm's accuracy.

### 3.1 Algorithm Improvement

The accuracy of the SPE reconstruction is dependent on the accuracy of the seed. Given a seed that is inaccurate by 6° or more, SPE typically cannot recover, and produces a reconstruction that is inaccurate by 6° or more. In addition, the likelihood space for SPE can contain multiple local maxima, so improving the accuracy of a seed already near the true solution will improve the accuracy of SPE. Thus, we focused our work on improving the quality of the seed.

As the muon travels though the detector, it generates hits. As indicated in Equation 1, linefit fits a line to these hits, weighting each hit quadratically in its distance from this line. This quadratic weighting makes the model sensitive to outliers. There are two reasons why outliers may appear far from the muon track:

1. Some of the photons can scatter in the ice for over a microsecond, which means that when they are recorded by a PMT, the muon will be over 300 m away.
2. While the noise reduction filters remove most of the outlier noise, the noise hits that survive can be far from the muon.

Our solution to the outlier problem was twofold: improve the modeling of the scattering and replace the least-squares optimization problem with a robust line-fitting algorithm.

#### 3.1.1 Improving the Scattering Model

The least-squares model does not model the scattering. Thus, hits generated by photons that scattered for a significant length of time are not useful predictors of the muon's position. We found that a filter could identify these scattered hits, and improve accuracy by almost a factor of two by filtering them from the dataset.

A hit $(\vec{x}_i, t_i)$ is considered a scattered hit if there exists a neighboring hit $(\vec{x}_j, t_j)$ that is within a distance of $r$ and has a time coordinate that is $t$ earlier than $t_i$. If $(\vec{x}_i, t_i)$ is a scattered hit, it is filtered out.

More formally, let H be the set of all hits for a particular event. Then, we define the scattered hits as

$$\left\{(\vec{x}_i,t_i) \mid \exists(\vec{x}_j,t_j) \in H.\ \left\|\vec{x}_i - \vec{x}_j\right\|_2 \le r \text{ and } t_i - t_j \ge t\right\}. \tag{3}$$

Optimal values of $r$ and $t$ were found to be 156 m and 778 ns, respectively, by parameter search.

#### 3.1.2 Adding Robustness to Noise

As described in Section 2.3, the least-squares model gives all hits quadratic weight, whereas we would like to limit the weight of the outliers. Some models in classical statistics marginalize the weight of outliers. We experimented with replacing the least-squares model with a variety of robust models: a deadzone-linear fit, a one-norm fit, and a Huber fit [3].

Of the models that we tested, the Huber penalty function gave the greatest increase in reconstruction accuracy. More formally, we replace Equation 1 with the optimization problem:

$$\min_{t_0,\vec{x}_0,\vec{v}} \sum_{i=1}^{N} \phi(\rho_i(t_0,\vec{x}_0,\vec{v})), \tag{4}$$

where the Huber penalty function $\phi(\rho)$ is defined as

$$\phi(\rho) \equiv \begin{cases} \rho^2 & \text{if } \rho < \mu \\ \mu(2\rho - \mu) & \text{if } \rho \ge \mu \end{cases}. \tag{5}$$

Here, $\rho_i(t_0,\vec{x},\vec{v})$ is defined in Equation 2 and $\mu$ is a constant calibrated to the data (for this application, the optimal value of $\mu$ is 153 m).

The Huber penalty function has two regimes. In the near-hit regime ($\rho < \mu$), hits are assumed to be strongly

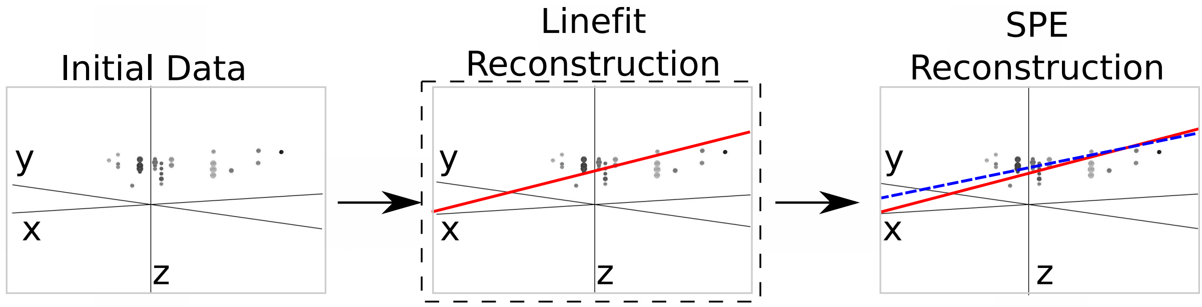


**Fig. 2**: The reconstruction pipeline used to process data in the IceCube detector. Each point indicates a PMT that recorded a photon (PMTs that recorded nothing are omitted). After initial data is collected and passed though some noise filters, the data is processed by a linefit (solid line), which is used as the seed for the SPE (dashed line). The SPE reconstruction is then evaluated as a potential neutrino. Our work in the reconstruction problem makes changes to the linefit reconstruction algorithm (indicated by the dashed box).

correlated with the muon's track, and the Huber penalty function behaves like least squares, giving point quadratic weight. In the far-hit regime ($\rho \geq \mu$), the Huber penalty function gives points linear weight, as they are more likely to be noise.

In addition to its attractive robustness properties, the Huber fit's weight assignment also has the added benefit that it inherently labels points as outliers (those with $\rho \geq \mu$). Thus, once the Huber fit is computed, we can go one step farther and simply remove the labeled outliers from the dataset. A better fit is then obtained by computing the least-squares fit on the data with the outliers removed.

#### 3.1.3 Implementation

Our scattering filter has a worst-case complexity that is quadratic in the number of PMTs that recorded a hit, but this is typically only between 10 and 100 PMTs. Unlike linefit, the Huber regression does not have a closed form solution, and thus must be solved iteratively. We use an alternating direction method of multipliers [2] to implement the Huber regression.

### 3.2 Results

We now present our empirical results, which validate our changes to the linefit. We also present our runtime performance results.

#### 3.2.1 Accuracy Improvement

Our goal is to improve the accuracy of the reconstruction in order to better separate neutrinos from cosmic rays. Thus we present three measurements: (1) the accuracy change between linefit and the new algorithm, (2) the accuracy change when SPE is seeded with the new algorithm, and (3) the improvement in separation between neutrinos and cosmic rays.

To measure the accuracy improvement, we use the metric of *median angular resolution* $\theta_{med}$, which is the arc distance between the reconstruction and the true track. Our dataset is drawn from simulated neutrino data designed to be similar to that observed at the Pole. We find that we can improve the median angular resolution of the simple reconstruction by 57.6%, as shown in Table 1.

We also find that seeding SPE with the improved simple reconstruction generates an improvement in the angular resolution of 12.9%, and that these improvements in the reconstruction algorithm result in 10% fewer atmospheric muons erroneously reconstructed as up-going, and 1% more muons correctly reconstructed as up-going.

**Table 1**: Median angular resolution (degrees) for reconstruction improvements. The first line is the accuracy of the prior least-squares model, and the subsequent lines are the accuracy measurements from cumulatively adding improvements into the simple reconstruction algorithm.

| Algorithm | $\theta_{med}$ |
|---|---|
| Linefit Reconstruction (Least-Squares) | 9.917 |
| With Addition of Scattering Filter | 5.205 |
| With Addition of Huber Regression | 4.672 |
| With Addition of Outlier Removal | 4.211 |

#### 3.2.2 Runtime Performance

We now report the runtime of our implementation, which is written in C++. The individual mean runtime of each component of the new algorithm is presented in Table 2. As shown, our new algorithm is more computationally intensive than linefit, but only by approximately a factor of six.

**Table 2**: The mean runtime for each component of the new simple reconstruction, contrasted with the mean runtime of the original linefit. As shown, the total runtime is approximately six times that of the original linefit.

| Algorithm | Runtime ($\mu s$) |
|---|---|
| Linefit Reconstruction (Least-Squares) | 24.2 |
| Scattering Filter | 56.6 |
| Huber Regression | 47.5 |
| Outlier Removal | 51.8 |

## 4 Conclusions

Muon track detection is a challenging problem in the IceCube detector. We achieve a 13% improvement in reconstruction accuracy with the addition of a scattering filter, and a more robust line-fitting algorithm. We also achieve these results with a reconstruction algorithm that is only 6 times slower than linefit. Our reconstruction software runs on-site, and is included in all IceCube analyses.

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